In The Name of Allah





Abstracts

# The 3<sup>rd</sup> Workshop on Copula and Various Types of Dependencies

Department of Statistics, Faculty of Mathematics and Computer, Shahid Bahonar University of Kerman, Iran

Ordered and Spatial Data Center of Excellence, Ferdowsi University of Mashhad, Iran

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### Topics of the Workshop:

- History of Copula
- Construction of Copula
- Copulas and Dependence Concepts
- Modelling using Copula
- Applications of Copula in Financial Mathematics
- Copula and Spatial Statistics
- Software Concepts of Copula

# Preface

On behalf of the organizing and scientific committees, we would like to extend a very warm welcome to all the participants of the 3rd workshop on copula and various types of dependencies. We hope this workshop provides an environment for useful discussion and exchange of scientific ideas.

We wish to express our gratitude to the numerous individuals and organizations that have contributed to the success of this workshop, in which more than 40 colleagues, researchers, and postgraduate students have participated.

Finally, we would like to extend our sincere gratitude to the students of the Department of Statistics at Shahid Bahonar University of Kerman for their unstinting cooperation.

Department of Statistics Faculty of Mathematics and Computer Shahid Bahonar University of Kerman Ordered and Spatial Data Center of Excellence Ferdowsi University of Mashhad

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# Extended progressively type-II censored order statistics

# Afhami, B.\*and Madadi, M.

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#### Abstract

In this paper, we study the extended progressively type-II censored order statistics arising from a random vector with identical but dependent components. An Archimedean copula with completely monotone generator is used to consider dependency of random vector.

 $<sup>0^{*}</sup>$  Speaker.





# Archimedean copulas: constructions and properties

Amini, M.\*and Mohtashami Borzadaran, H. A.

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#### Abstract

Archimedean copulas are one of the most popular models used in finance, hydrology, etc. The construction of these models are of a great importance due to their flexible dependence structure. Several methods of producing these families have been studied such as Marshall-Olkin, Williamson-d-transform, frailty models, utility functions, Laplace transform and distortion function. In this article we study these construction methods and reveal their properties in bivariate and multivariate cases by considering the fact that these methods may intersect.

<sup>&</sup>lt;sup>0</sup><sup>\*</sup> Speaker.





# $\begin{array}{c} \textbf{Consecutive-}k\textbf{-out-of-}n \text{ systems with} \\ \textbf{non-identical and dependent component} \\ \textbf{lifetimes subject to stochastic} \\ \textbf{deterioration} \end{array}$

### Amirzadeh, V.

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#### Abstract

In this paper, we consider linear and circular consecutive k-out-of-n systems under stochastic deterioration. It is assumed that lifetimes of the components are dependent and non-identically distributed according to an Archimedean copula and random deterioration in the resistance of a component is defined by a stochastic process. Illustrative examples are also provided.





# Copula-based measures of asymmetry: a review and recent results

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#### Abstract

Let (X, Y) be a pair of continuous random variables with the joint distribution function  $H(x, y) = P(X \le x, Y \le y)$ , univariate marginal distributions  $F(x) = P(X \le x)$ ,  $G(y) = P(Y \le y)$ , at each  $x, y \in \mathbb{R}$ . Let C be the unique copula associated with (X, Y) through the relation

$$H(x,y) = C\{F(x), G(y)\}, \quad x, y \in \mathbb{R},$$

in view of Sklar's Theorem Sklar (1959). In fact, C is the cumulative distribution function of the pair (U, V) = (F(X), G(Y)) of uniform (0,1) random variables. A key concept in statistical literature is the concept of symmetry. One can define symmetry in terms of structural properties of the distribution function. Even though there is a unique way of defining symmetry in one dimension, there exist more than one definition in the multidimensional case. Nelsen (1993) examines various definitions of bivariate symmetry. There are some tests for identifying symmetry in the statistical literature. However, little effort was made in proposing measures for evaluating the degree asymmetry present in data. This talk will review the recent investigations on measuring the degree of asymmetry. For a given C, let  $\hat{C}(u, v) =$ u + v - 1 + C(1 - u, 1 - v) be the survival copula or reflected copula associated with C or equivalently, the cumulative distribution function of the pair (1 - U, 1 - V). A copula C is said to be radially symmetric





Nelsen (1993) if

$$C(u,v) = \widehat{C}(u,v), \quad \text{for all } u,v \in [0,1]. \tag{0.1}$$

This concept is also called 'reflection symmetry' and 'tail symmetry' in literature, see, e.g. Nelsen (1993). When the condition (0.1) fails for some  $u, v \in [0, 1]$ , the copula C is said to be radially asymmetric. Any suitably normalized distance between the surfaces z = C(u, v)and  $z = \widehat{C}(v, u)$ , in particular, any  $L_p$  distance, will yield al measure of radial asymmetry. Diagnostics such as asymmetry measures would be useful during data analysis. For instance the presence of radial asymmetry in a set of data rejects the null hypothesis of bivariate normality and other models with more flexibility should be considered. Recently several copula-based measures of radial asymmetry and the desirable properties for such measures addressed in Alikhani et.al. (2015). Some tests for identifying radial symmetry of bivariate copulas discussed in Buzebda and Cherfi (2012). This kind of asymmetry distinguished from the issue of whether a copula is exchangeable, i.e., that for all  $u, v \in [0, 1], C(u, v) = C(v, u)$ . For discussion on this kind of symmetry we refer to Bacigál et.al. (2001). The purpose of this talk is to review the recent investigations on measuring the degree of bivariate asymmetry.





# Application of copula in groundwater quality interpolation

### Ganjalikhani, M.\*1, Zounemat-Kermani, M.1, Rahnama, M. B.1 and Rezapour, M.2

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#### Abstract

This study presents a new method for interpolation by using copula for groundwater quality zoning. For this purpose the qualitative data of 107 observation wells in Ravar and Kerman plain contains Bicarbonate, Sulphate, Calcium and Total Dissolved Solids (TDS) at winter of 2014 were used. Then, the obtained results were compared to the results obtained from conventional zoning methods to evaluate the performance of copulas. Analysis of the results with respect to the root mean square error showed that copula has a higher ability than common methods in qualitative zoning of groundwater resources.

 $<sup>0\</sup>ast$  Speaker.





### Non-linear regression under progressively type-II censored order statistics with dependent components

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#### Abstract

In this paper, we obtain non-linear regression for progressively type-II censored order statistics arising from dependent units that are jointly distributed according to an Archimedean copula. An illustrative example that consider the Clayton family is also provided.

 $<sup>0^{*}</sup>$  Speaker.





### The behavior of non-identical and dependent standby components in a coherent system

# Khosravi, M.<sup>1\*</sup>, Pourmousa, R.<sup>1</sup>, and Rezapour, M.<sup>1</sup>

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#### Abstract

In this paper, a coherent system consisting of non-identical dependent active components and equipped with non-identical dependent standby components is considered. The main object of this study is the random quantity which account the number of surviving standby components when the system is failed. We represent the distribution of the corresponding random variable in terms of system signature.

<sup>&</sup>lt;sup>0</sup><sup>\*</sup> Speaker.





# Properties of Csiszar $\phi$ -divergence and links via copula function

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#### Abstract

After the invention of the Shannon 1948 the extension of notion for the measure of information, with application in communication theory have been obtained. With the paper of Kullback and Leibler (1951) and their book in 1955 have raised the link between statistical inference and information theory with the applications in various fields. The Kullback-Leibler (KL) information is a fundamental quantity in probability and statistics, that measures the similarity of two distributions. Many divergence such as Renvi, Hellinger,  $\chi^2$  divergence, Tsallis and Bhattacharvya distance have attracted the attention of many researches for various interpretations in these divergences have been applied to discovering the phenomena in the world via the KL information idea. We have surveyed the generalization of entropy and divergences between two probability density functions (or pmfs). Csiszar (1963) defined a general divergence that subsumes lots of well-known measures. We discussed and reviewed some of properties of this divergence, and also most of the special cases. Here, we have concentrated mostly on bivariate distributions. The mutual information can be expressed as a distance to the statistical independence in the space of distribution measured by the KL information between the actual joint

 $<sup>0\</sup>ast$  Speaker.





distribution and the product of two marginal. Basic general properties of  $\phi$ -divergences, including their axiomatic, and some important classes of  $\phi$ -divergences are derived via the distance between the joint distribution and its product of marginal. This can be related to dependence. Minimization of this divergence under constraints is also a direction of this work. Copula constructed the bivariate (or multivariate) distribution function based on a function of marginal distribution functions that was introduced by Sklar (1959). Connections between the above results and its alternative via copula is our interest in this paper. The copula entropy is the difference between joint entropy and sum of the marginal entropy is an important rule. Our attention will be on  $\phi$ -divergence based on copula entropy and dependence. The principle of maximum entropy provides a method to select the unknown pdf (or pmf) compatible to entropy under a specified constraint. This idea was introduced by Jaynes 1957 and obtained with a theorem by Kagan et al. (1973). Similar to arguments in minimization of Kullback Leibler information, we have applied to the minimization of  $\phi$ -divergence under specified constraints. Also, maximum copula entropy under constraint with simple example are achieved. Finding copula entropy in bivariate equilibrium distribution and inequality of  $\phi$ -divergence for weakly negative dependence via some distributions is the last part of the paper.





# Construction a family of nonseparable spatio-temporal covariances by using copula functions

### Omidi M. and Mohammadzadeh M.\*

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#### Abstract

Statistical analysis of natural phenomena with spatial and temporal correlations requires to specify their correlation structure via a covariance function. A separable spatio-temporal covariance function is usually used for the ease of application. Nonetheless, the separability of the spatio-temporal covariance function can be unrealistic in many settings, where it is required to use a non-separable spatio-temporal covariance function. In this paper, a structural copula function is applied to construct a family of non-separable spatio-temporal covariance function. Next, a modified genetic algorithm is applied to explore the spatio-temporal correlation structure of Ozone data in Tehran, Iran.

 $<sup>0\</sup>ast$  Speaker.





# A copula approach to bivariate credible confidence interval for frequency and severity of Bonus-Malus systems

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#### Abstract

The most of advanced Bonus-Malus systems works based upon both frequency and severity of reported claims. This article utilizes copula idea to develop a bivariate credible confidence interval for frequency and severity of a given Bonus-Malus system.

<sup>&</sup>lt;sup>0</sup>\* Speaker.





# The most common goodness of fit tests for copula models

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#### Abstract

Copulae is one of the main ways of modelling dependence. Many proposals have been made recently for goodness-of-fit testing of copula models. In this paper we propose and analysis the several most common methods of goodness of fit test that use for copula selection . We eventually apply these methods to select a suitable copula of the two variables associated with the Iran's financial data: gross domestic production, oil income index.

 $<sup>0^{*}</sup>$  Speaker.





# On construction of nested Archimedean copulas

### Rezapour, M

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#### Abstract

McNeil (2008) gave certain conditions under which a nested Archimedean copula is indeed a copula. Here, following the work of McNeil and Nešlehová (2009), we present some weaker conditions under which a partially nested Archimedean copula with two nesting levels is still a copula. We also obtain the density function of partially nested Archimedean copula C with two nesting levels and  $d_0$  child copulas and state certain conditions under which a partially nested Archimedean copula with arbitrary nesting levels is indeed a copula. Various examples including famous Archimedean families and transformations of such are given.